A Quick Method for Analyzing Hartmann-Shack Patterns: Application to Refractive Surgery

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ABSTRACT

A quick method to analyze the Hartmann-Shack pattern was developed. This method allows us to study the limitations of the system and to determine how to improve its capacity to measure a large range of aberration. It can treat situations presenting difficulty when analyzing a detected pattern. An application to eyes with large amounts of aberration due to corneal surgery is considered. [*J Refract Surg* 2000;16:S636-S642]

INTRODUCTION

An optical system provides a perfect formation of a point image if all image-forming rays meet in a single point. However, such an ideal condition is never fulfilled in practice. Like any optical system, the eye suffers from failure to conform to the mode of ideal image formation.[1] Departures from these conditions are referred to as aberrations.[2] The more the aberration increases, the more the image differs from the original object, and therefore, the more the quality of the image formed on the retina is degraded. In this study, we limit our attention to geometrical aberrations.

For a perfect aberration-free eye, the wavefront reflected by the retina and observed at the entrance pupil is strictly a plane wave. Departures from this reference distribution build the wavefront aberration (Fig 1). Several methods have been used to measure monochromatic aberrations.[3,4] In this study, we focused on the Hartmann-Shack method[5,6] given that it is an objective, rapid, and precise technique. After the objective measurement, a numerical effort is required to derive the profile of aberration. The most used calculating method is based on wavefront fit.[7] We propose a new method referred to as a direct method for calculating the profile of an aberration measured by a Hartmann-Shack sensor. It allows rapid calculation of the aberration profile. In addition, this method produces a qualitative interpretation of the aberration profile at first sight. We show how this method can handle extreme cases of poor image quality such as the case of operated eyes.

HARTMANN-SHACK APPARATUS

Aberration can be measured by analyzing the wavefront reflected by the retina and observed at the entrance pupil. The principle of the Hartmann-Shack apparatus consists of dividing the wavefront into a set of elementary segments with different slopes (Fig 2). A lenslet array is used to measure these slopes. In the focal plane, the lenslet array provides an array of bright spots. The spots are uniformly distributed (constant spacing) for the case of an aberration-free eye. The uniform pattern is then called a reference pattern. For a deformed wavefront, each spot is laterally shifted from the axis of the corresponding lenslet by an amount increasing with the slope. Let us consider only one lenslet. The local shift observed in the focal plane of this lenslet is expressed as follows :

$$\begin{cases} dx' = f \bullet \dfrac{\partial W(x,y)}{\partial x} \\ dy' = f \bullet \dfrac{\partial W(x,y)}{\partial y} \end{cases} \tag{1}$$

where f is the focal length of the lenslet array, dx' and dy' are the lateral shifts of the spot corresponding to one lenslet, and $W(x,y)$ is the aberration at a point (x,y). We observe an irregular array of spots in the focal plane, at a distance f behind the lenslet array.[5] During the experiment, ametropic eyes must be corrected. For instance, if the eye is an aberration-free myopic eye, we may obtain no useful spots in the focal plane due to defocus. For operated eyes, if the non-operated zone is highly myopic, then the lenslet array generates no spots at the distance f.

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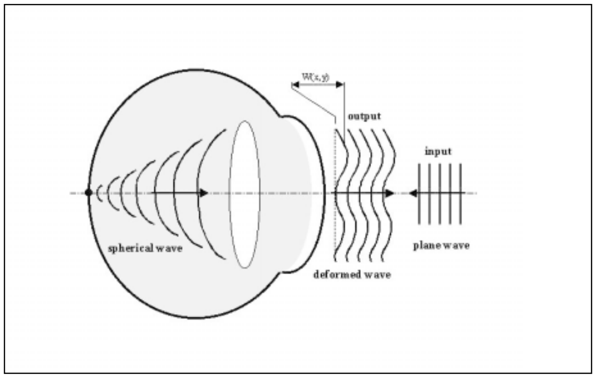

Figure 1. The eye is illuminated by a plane wave. For an ideal eye, the wave traveling inside this organ is a spherical wave that provides a bright point on the fovea. In reality, the wavefront is not spherical. If the fovea acts as a point source, then it provides a divergent spherical wave that exits the eye as a plane wave. In reality, we obtain a deformed wave.

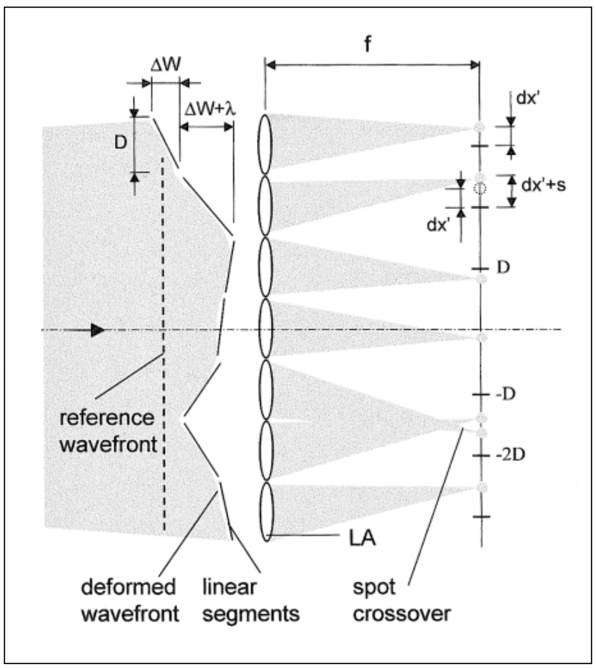

Figure 2. Wavefront is divided into four linear segments and local slopes are determined by means of a lenslet array (LA), f: focal length; D: lenslet size; dx': local shift; s: spot size; ΔW: aberration corresponding to dx'.

For this zone, because of defocus the lenslet array generates a large pattern that overlaps with the spots corresponding to the operated (corrected) zone. Thus, we observe a very noisy pattern in the focal plane.

The basic Hartmann-Shack measurement method has been improved.[6] A brief description of the setup is presented in Figure 3. A monochromatic He-Ne laser source is used to illuminate the sub-

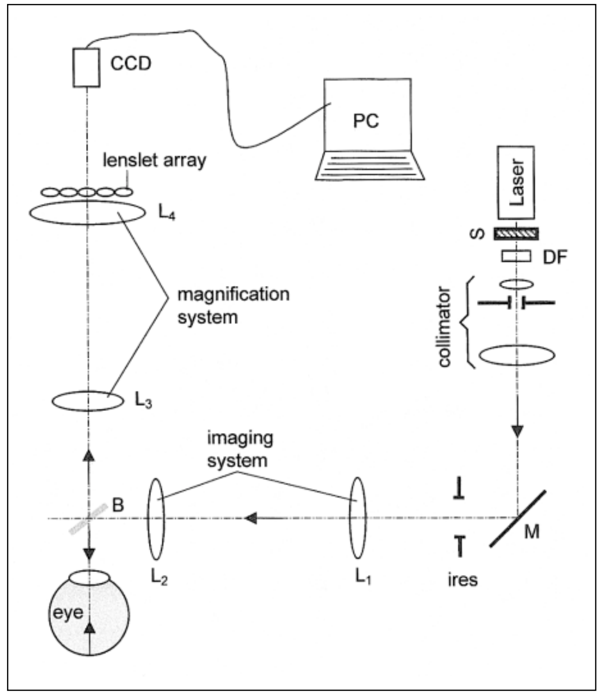

Figure 3. Setup for the measurement of the aberrations of the human eye. A laser, emitting light in the visible domain, is used for illumination. B, beam splitter; DF, density filter; S, electronic shutter, L_1 - L_4, convex lenses; M, mirror; LA, lenslet array.

ject's eye. The beam travels through a shutter, S, that controls the exposure time, and a neutral density filter, DF, that is used to adjust the power to eye-safe levels. After being expanded by a collimator, the wavefront at the level of the iris is imaged by means of two lenses onto the pupil of the eye to control the size of the illuminating beam. Light travels through the optics of the eye twice. The diffusing reflection of light from the retina fills the whole pupil. The reflected wavefront to be measured is laterally magnified and finally transformed into a matrix of spots by the lenslet array. The lateral shifts of these spots are then used to numerically reconstruct the wavefront aberration.

CALCULATION OF THE ABERRATION

Using the measured slopes, the profile of the wavefront aberration can be performed by a wavefront fit.[7] The objective is to obtain an explicit analytical form of the aberration: $W(x,y)$. The method consists of fitting the best possible expression of $W(x,y)$, the partial derivatives of which will satisfy equation (1). In other words, the technique consists of minimizing the difference between the derivatives of the analytical function $W(x,y)$ and the measured local shifts. The wavefront estimation can be made by first choosing a suitable polynomial expression for the wavefront and using a least squares estimating routine to estimate the best coefficients for each polynomial term.[7]

We propose an alternative based on the analysis of the increase of aberration when the measured spot is shifted by one spot size of the reference pattern (Fig 2). This approach is referred to as a direct method for calculating the profile of aberration measured by the Hartmann-Shack sensor. For simplicity of notation, we restrict our attention to one dimension—the x axis.

A square lenslet, with a focal length f and a diameter D, that is illuminated by a plane wave provides a spot with a size, s, that we refer to as the reference spot size:

$$s = \lambda f \ / \ D = \lambda F\# \qquad (2)$$

due to diffraction.[8] The F-number : $F\# = f \ / \ D$ is defined as the ratio of the focal length to the diameter of the lens. This reference spot size s corresponds to the semidiameter of the Airy disk of the diffraction pattern in the focal plane of the square lenslet. For a deformed wavefront, the spots are expected to be larger than the size expressed in Eq. 2 because of the appearance of high spatial frequencies. The measured spot, that corresponds to a deformed wave (Fig 2), will be shifted by:

$$dx' = f \ \Delta W / D = \Delta W \ F\# \qquad (3)$$

where ΔW is the maximal change in wavefront aberration across the segment corresponding to this lenslet. If the spot further moves by a reference spot size ($dx'+s$ instead of dx'), then by using equation (3), the amount of aberration will be $\Delta W+\lambda$ instead of ΔW (Fig 2). Because of Eqs (2) and (3), the following relation is valid :

$$dx' + s = (\Delta W + \lambda) \ F\# = f \ (\Delta W + \lambda) \ / \ D \qquad (4)$$

This result states that a spot shift of one reference spot size implies an increase (or decrease) of aberration by one wavelength. This implies that at first sight one can compare measured patterns with each other. One can even have a visual estimation of the maximal value of aberration before calculating the aberration profile. This result can also be used to determine the accuracy of the measurement. For example, if we are able to measure a shift of one-fifth of the reference spot size, then the apparatus permits measurement of an aberration with an accuracy (resolution) of one-fifth of the wavelength.

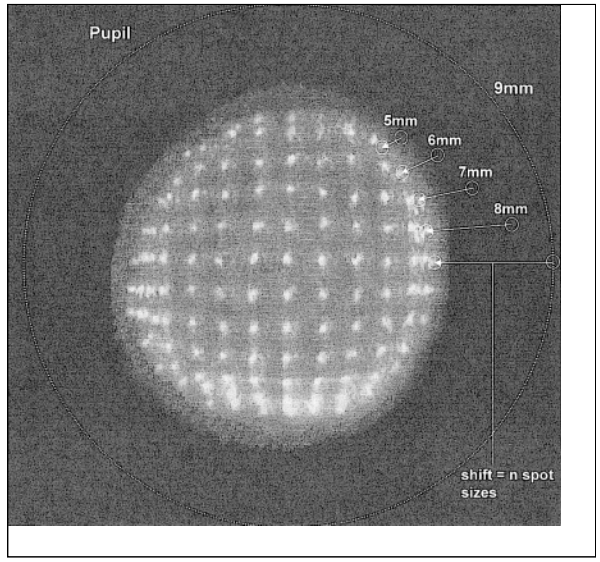

Figure 4. Estimation of the maximal value of aberration for a PRK pattern. If we know the value of the maximal value for an 8-mm pupil, we can deduce a lower band of the maximal value of aberration for a 9-mm pupil.

By displaying both the reference and the detected pattern, one can globally estimate the retinal image quality. One should attribute a variation of one λ of the aberration for each displacement of one reference spot size with respect to the reference pattern. Therefore, we can plot directly the aberration curve.

ANALYZING THE LIMITS OF THE SETUP BY THE DIRECT METHOD

Maximal Measurable Value of Aberration

The maximal value of aberration that can be measured by one lenslet corresponds to a spot shift of half the spot spacing of the reference pattern, $D/2$ (on the left or on the right). If the aberration value is larger, the spot crosses over into an area where a neighboring spot is expected (Fig 2). Thus, if no spot crossover occurs, the maximal spot shift is $0.5 \ D/s$ reference spot sizes ($dx' = 0.5 \ D$). The maximal measurable value of aberration corresponding to one lenslet is then $0.5 \ D/s$ wavelengths. One of the characteristics of the lenslet is its compression ratio Rc, which is defined as the ratio of spot spacing to spot size ($Rc = D/s$). The maximal value of aberration measured by a lenslet is then $0.5 \ Rc$ wavelengths. If both dimensions (x and y) are considered, a multiplicative factor of square root of 2 is required. If we are sure that, in spite of spot crossover, no spot overlap will occur (in the case of large spherical aberration), then the maximal value of aberration measured by one lenslet is larger than *0.5 Rc* wavelengths. Spot crossover results in spot overlap if either the aberration is extremely large or if the slope of the aberration curve is steep and the direction of the slope is abruptly inverted (Fig 2). If for a given pupil diameter P, N lenslets are illuminated ($P=N\ D$), the maximal measurable value of aberration is : $0.5\ N\ Rc\ \lambda = 0.5\ P\ /\ F\#$. The maximal measurable value of aberration does not depend on the wavelength used for the measurement. For a given pupil size P, it increases with the lenslet size D and decreases with the focal length f. In other words, it decreases with the $F\#$, which presents a characteristic of the lenslet array. For a given lenslet array, the maximal measurable value of aberration increases with the number N of illuminated lenslets. Thus, it is advantageous to magnify laterally the wavefront before its analysis through the lenslet array.

In practice, a compromise must be made because of the following reason. Bearing in mind relation (2), decreasing the $F\#$ is advantageous while the reference spot size is still larger than the smallest feature size of the CCD camera. The spot must not be undersampled, otherwise the determination of the spot shift will be not accurate. Each spot must be represented by at least three pixels in each direction. To determine the spot shift one should first determine the centroid of spot. For a pixel size p of the camera, one can determine the resolution of the measurement. In other words one can determine the smallest amount of aberration that can be measured. Let us express the pixel size of the camera in terms of the reference spot size $p = (r\ /\ \lambda)$ s where r is a real factor, that we call resolution and which is smaller than 1. The aberration amount corresponding to a shift of one pixel is : $(p\ /\ s)\ \lambda = r$. For example, if the size s of a reference spot is composed of 5 pixels, then the resolution of the measurement is $r = 0.2\ \lambda$. Athough decreasing the $F\#$ is advantageous in terms of the maximal measurable aberration, it is unfortunately disadvantageous in terms of resolution.

How To Choose the Lenslet Array

The direct method facilitates the choice of system parameters. In this analysis, we limit attention to the choice of the lenslet array and the camera. Let us consider three epoxy on glass lenslet arrays produced by the Adaptive Optics Associates (Table). The maximal measurable value of aberration is calculated for the case of no spot crossover by means of the direct method for calculating the aberration according to section 4 (λ = 0.632 µm).

Table
Three Models of Epoxy on Glass Lenslet Arrays

Parameter	Model 1060-260-S	Model 0400-53-S	Model 0600-40-S
Focal length	260 mm	53 mm	40 mm
Lenslet pitch (spot spacing)	1.06 mm	0.4 mm	0.6 mm
F-number	245	132.5	66.6
Compression ratio	6.8	4.7	14.28
Reference spot size	155 µm	84 µm	42 µm
Number of lenslets	24x24	60x60	45x45
Max. value of aberrations for a pupil of 9 mm	29 λ	54 λ	107 λ

From the Table, we note that the most advantageous lenslet array in terms of limitations of the system (maximal measurable value of aberration) is the 0600-40-S model. Compared to the 1060-260-S model, the maximal measurable value of aberration of the 0600-40-S model for a pupil of 9 mm is 367% bigger. In addition, a measured pattern is represented by more spots (177% = 1.06/0.6) than for the case of the 1060-260-S model. The drawback of 0600-40-S model is the fact that the reference spot size is small. The 0400-53-S model is more advantageous in terms of reference spot size. The smallest feature size of the CCD camera should be significantly smaller than the reference spot size, otherwise the calculation of the centroid of the spot is not accurate. This calculation is necessary to determine the local shift. Given that the reference spot size s is equal to 42 µm for 0600-40-S model, the pixel size p of the camera must be 14 µm or less (the spot is represented by at least three pixels). The main drawback of the 0400-53-S model is that the compression ratio is small. This affects the accuracy of the measurement. Because the reference spot size is almost one quarter of the spot spacing, the uncertainty of calculating the spot shift is relatively big. If for any reason (speckles, etc.) we made an error of $s/2$ when calculating the local shift, the resulting amount of aberration ($\lambda/2$) corresponds to one-eighth of what the lenslet can measure. Such an error may be made by other lenslets yielding to a relatively big error and consequently to an erroneous measurement. From this point of view, the 0600-40-S model is the most advantageous.

Let us determine the resolution of the three models. We need to calculate the value of r : the smallest amount of aberration that can be measured by each of the three models. Let us suppose that the pixel size of the camera is $p = 12\ \mu m$. For the model 0600-40-S, one pixel corresponds to 28.5% of the

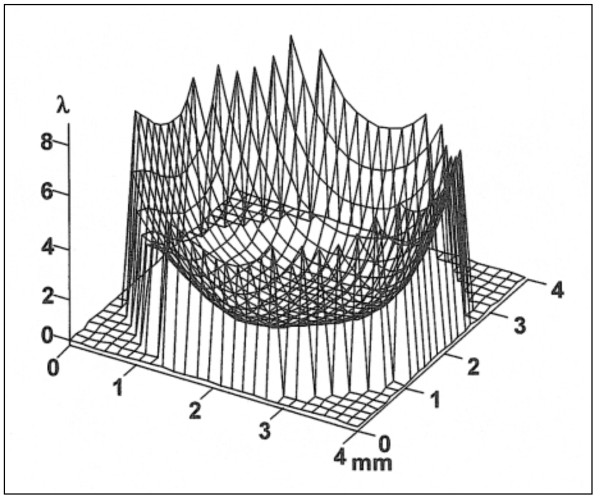

Figure 5. The aberration profile calculated for the central zone (4 mm) of the pattern in Figure 4.

reference spot size ($p = 0.285\ s$). Thus, the resolution of the measurement is $r = 0.285\ \lambda$. For the models 0400-53-S and 1060-260-S, we find respectively the resolutions of 0.143 λ and 0.077 λ. Thus, the 1060-260-S model is the most advantageous in terms of resolution of the aberration measurement.

REFRACTIVE SURGERY

The amount of the aberration may be so significant that some spots are shifted into areas where other spots are expected (Fig 2). The difficulty is that in general we cannot strictly know the origin (lenslet) of each spot (which lenslet provides which spot). One has the tendency to assume that the upper spot corresponds to the upper lenslet and the lower spot corresponds to the lower lenslet. This is, however, not valid for all lenslets of Figure 2. To illustrate the usefulness of the direct method for analyzing the Hartmann-Shack pattern, we consider the case of corneal surgery. We limit attention to the calculation of the maximal value of aberration. A study of the optical effects of corneal surgery (in particular photorefractive surgery) has been the subject of a previous work[9] and it goes beyond the scope of the present work.

It is expected that the operated cornea will have a more degraded optical performance than its preoperative counterpart.[9,10,11] The main reason for the degradation of image quality for a large pupil is the fact that the operated eye possesses two different optics with a transition zone between them.[9] For instance, if the pupil of an initially myopic eye is widely opened, then the operated zone generates an in-focus image in the ideal case. However, the non-operated zone, which is not corrected (myopic), generates a defocused image that is superimposed with the in-focus image on the retina. The retinal image is then drastically deformed (aberrated). The amount of aberration is expected to increase with the required correction for the non-operated zone. In addition, the transition zone, that is also not exactly corrected, further affects the image quality.[9]

The technique of measurement consists of calculating the aberration profile with the direct method or even with the method of the best wavefront fit for the central zone where no spot crossover occurs. The rest of the pattern is ignored during the calculation. We can then calculate the maximal value of aberration at the periphery of this zone. For the rest of the measured pattern, we estimate the maximal value of aberration for each pupil size as follows. Knowing the pupil size, we estimate how much the spots of the outer zone have moved. We assume that these spots have moved until the zone of blur (Figs 4 and 7). We consider the blur to be the result of spot overlap. It is theoretically possible that the outer spots move into the central zone. Because of possible high frequencies in the outer zone, we see no spot but do see a spread blur. This blur, however, does not exist in the outer zone of the measured pattern. It can only be in the central zone or in the ring of blur surrounding it (Figs 4 and 7). Thus, if we assume that the outer spots have moved until the zone of blur, we obtain a lower band of the maximal value of aberration because the spot may have moved further. In Figure 4, the pattern is measured for a pupil dilated to a diameter of 9 mm. Let us suppose that we know the maximal value of aberration W_8 for a pupil of 8 mm. We know that the outermost spot has moved at least by n spots sizes (n: real number), which implies an increase of aberration of n. We can then deduce a lower band for the maximal value of aberration corresponding to a pupil of 9 mm : $W_9 \geqslant W_8 + n\ \lambda$.

The central zone of no spot overlap extends over a diameter of 4 mm. For this zone, we can calculate the aberration profile by either the direct method or the method of the best wavefront fit (Fig 5). Then, we can estimate a lower band of the maximal value of aberration for each of the pupil sizes : 5 mm, 6 mm, 7 mm, 8 mm, and 9 mm. Another example of corneal surgery is presented in Figure 7.

ILLUSTRATIONS

Figure 6 illustrates an example of a measured pattern and its corresponding aberration profile calculated with the direct method. The positions of the reference spots are indicated by crosses on the

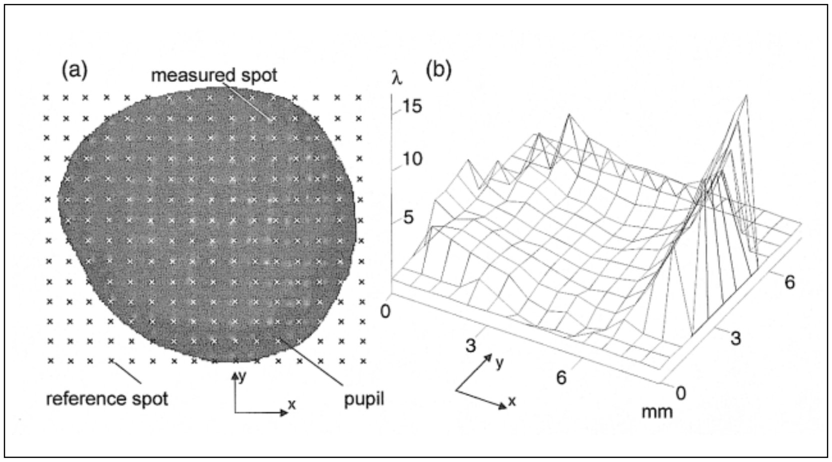

Figure 6. a) Measured pattern and reference spots. Reference spots are indicated by crosses. **(b)** Aberration profile.

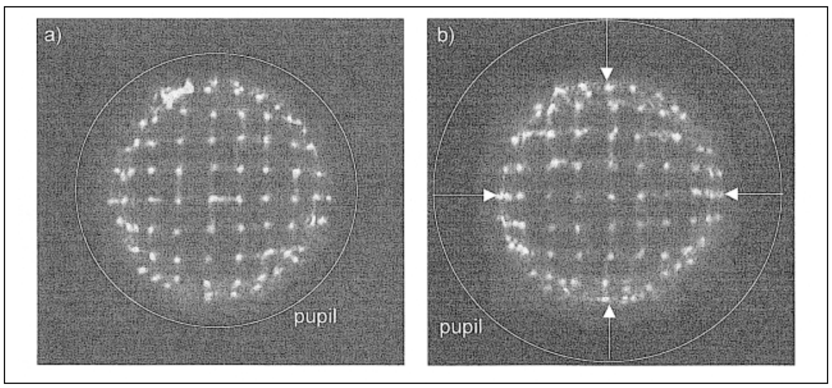

Figure 7. Spot shift for corneal surgery patients. **a)** Before dilation (7 mm), and **b)** after dilation (9 mm).

detected pattern (Fig 6a). On the right side of the detected pattern the spots are largely shifted compared to the left side or to the central zone. This kind of asymmetrical pattern is observed for coma. Figure 6a also shows that lateral shifts along the y-axis are smaller compared to those along the x-axis. At first sight of the detected pattern, we can speculate that the examined eye is likely suffering from x-axis coma. Given that on the left side of the pattern the spots do not diverge away from the central zone, other types of aberration (especially irregular aberration) are involved in addition to coma. Because shifts along the y-axis are small, the eye is unlikely to be suffering from spherical aberration. This qualitative interpretation is confirmed by Figure 6b.

To illustrate how the direct method can handle the case of large aberration and spot crossover, we considered an example of photorefractive surgery.[9] The patient was a 26-year-old woman with initial myopia of -4.00 D in her right eye. Aberration was measured for the eye 2 years after surgery (visual acuity after surgery: 6/6, no correction required). Figure 7 shows the detected patterns before (7 mm) and after dilation (9 mm) of the eye. Both patterns have almost the same extent although the pupil sizes are different. Indeed, all spots are confined in a diameter smaller than 4.5 mm, given that the sampling interval is 0.5 mm. The central zone was analyzed by the wavefront fit method and also by the direct method. Both methods gave the same result. The maximal value of aberration was calculated for a pupil of 4 mm and we obtained $W_4 = 16.5\lambda$.

The arrows in Figure 7b show the shifts of the outermost spots. Because the outer spots moved radially into the central zone, one can state that positive spherical aberration dominates. The outermost spot has moved at least across 4 adjacent spots, yielding that the total shift is of more than $5\ Rc\ s$. In our experiment, Rc is equal to 6.2. By using the direct method, we can estimate a lower limit for the aberration at a pupil size of 9 mm. For a pupil size of 5 mm, because the outermost spot has moved by a distance of almost one spot spacing, the value of aberration is assumed to be bigger than $Rc\ \lambda + W_4 = 23\ \lambda \leq W_5$. For a pupil size of 7 mm, the outermost spot has moved by at least $3\ Rc\ s$, and we then obtain a lower limit for the amount of aberration that is equal to $W_7 \geqslant 3\ Rc + W_6 \geqslant 55\lambda$ ($W_6 \geqslant 2Rc + W_5$). For a pupil size of 9 mm we obtain $W_9 \geqslant 5Rc + W_8 \geqslant 110\lambda$.

DISCUSSION

A direct numerical method was developed to reconstruct rapidly the aberration profile from a set of measured spots, without passing through a wavefront fit. The method does not require precise knowledge of the position of the pupil center of the human eye. Without any computing effort, the user can estimate the maximal value of aberration and interpret the behavior of the wavefront aberration by displaying both the reference and the measured patterns. Moreover, the direct method allows analysis of the resolution (the smallest amount of aberration that can be measured) of the apparatus as well as the determination of the maximal measurable value of aberration. We can then determine which parameters need to be taken into account to increase the capacity of the apparatus to measure large amounts of aberration. In a similar way, we can determine which parameter to change in order to improve the resolution. We illustrated how the direct method can guide the user to choose the features of the lenslet array and the camera.

In general, extreme cases of poor image quality can be handled by the direct method. For some eyes suffering from large amounts of aberration, spot overlap can occur. This is likely to arise, for instance, in eyes with high ametropia[12] and in eyes following corneal surgery.[9] This situation presents difficulty when analyzing the detected pattern. To reduce the problem of spot overlap, we should shorten the focal length f and/or enlarge the lenslet spacing D. For a given lenslet array, for which D and f are fixed, a solution to avoid the problem of spot overlap consists of laterally magnifying the wavefront before illuminating the lenslet array. In this extreme case, in spite of spot overlap, a lower band of the maximal value of aberration can be given by means of the direct method.

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